Eur. Phys. J. D **18**, 347–351 (2002)
DOI: 10.1140/epjd/e20020040

THE EUROPEAN PHYSICAL JOURNAL D
EDP Sciences

Long-range perturbations produced by a localized packet of ion-acoustic waves in a collisionless plasma

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Received 25 July 2001 and Received in final form 31 October 2001

Abstract. A well localized packet of ion-acoustic waves propagating in a collisionless unmagnetized plasma produces long-range second-order perturbations of the moments of the electron distribution function due to those electrons which cross the region where the electric field is present, take a “kick”, and carry this information at large distances from the packet itself. An extended “halo” is thus formed around the wave-packet, with an intensity decreasing in space typically according to a power-law. This “halo” contains electrostatic as well as magnetic perturbations.

PACS. 52.35.-g Waves, oscillations, and instabilities in plasmas and intense beams – 52.38.-r Laser-plasma interactions – 94.20.Bb Wave propagation

1 Introduction

The excitation of low phase velocity wave-packets in collisionless plasmas can take place under various physical conditions, as for example by grid excitation [1–3] and in double-plasma devices [4], in low density plasmas, as a product of parametric instabilities induced by laser radiation in dense plasmas [5,6], and also in magnetized space plasmas [7]. Then the problem of their detection is of major concern and a better characterization of their interaction with the background plasma could help in developing new diagnostic methods for their observability.

In the recent years a renewed interest for the basic physic of ion-acoustic (i.a.) and ion-plasma waves has found its ground in the ability of doing accurate laser scattering measurements of small-scale plasma density inhomogeneities [8]. In this paper we discuss a new physical effect associated with the propagation of i.a. waves in an isotropic plasma, whose observation can be attempted by means of such sophisticated diagnostics. Consider a well-localized packet of i.a. waves propagating in an unmagnetized plasma. Does the presence of the wave-packet manifest itself at large distance from its location, where the amplitude of the electric field is exponentially small? In general terms the answer to this question is positive: as the motion of the electrons, rapidly crossing the wave-packet in all directions, is perturbed by the wave electric field, the electron distribution function (e.d.f.) turns out to be perturbed too, even far from the wave-packet. However, the linear part of the perturbation produces only exponentially small contributions to the moments of the e.d.f. (like, *e.g.*, the density) and, therefore, one cannot expect the presence of any noticeable electric and magnetic fields outside the packet. On the other hand, we demonstrate that the second order perturbations of the moments of the e.d.f. extend well beyond the area where the packet is localized; they are decreasing typically with a power law and then they lead to long range macroscopic effects.

This paper is organized as follows. In Section 2 the second order (in the field amplitude) perturbation of the electron distribution function is calculated for an i.a. wave-packet; Section 3 contains the derivation of the second order perturbations of the fluid moments. The long-range electrostatic and magnetic field perturbations are discussed in Section 4. Section 5 contains some concluding remarks.

2 Long range perturbation of the electron distribution function

Let us consider a weakly damped i.a. wave-packet $\varphi(\mathbf{r},t) = \psi(\mathbf{r},t)\exp(-\mathrm{i}\omega t + \mathrm{i}kz) + \mathrm{c.c.}$ propagating along the z-axis, where $\varphi(\mathbf{r},t)$ is the electrostatic potential of the i.a. wave-packet, $\psi(\mathbf{r},t)$ is its complex amplitude, slowly varying over the time $2\pi/\omega$, and almost uniform over the wavelength $2\pi/k$. Consistently, we assume the ion temperature much smaller than the electron temperature T [9]. The first order perturbation of the e.d.f. $f_1(\mathbf{r},\mathbf{v},t)$ is calculated by integrating the linearized Vlasov equation along the unperturbed electron trajectories; this gives

$$f_1(\mathbf{r},\mathbf{v},t) = \mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}kz}\int_{-\infty}^{t}\mathrm{d}t' g\left[\mathbf{r}+\mathbf{v}(t'-t),\mathbf{v},t\right] \times \mathrm{e}^{\mathrm{i}(kv_z-\omega)(t'-t)} + \mathrm{c.c.} \quad (1)$$

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where

$$g(\mathbf{r},\mathbf{v},t) = -\frac{\partial f_0}{\partial \mathbf{v}} \cdot \left(\mathrm{i}\mathbf{k}\psi + \frac{\partial \psi}{\partial \mathbf{r}} \right) \cdot \qquad (2)$$

The moments of equation (1) then read

$$M(\mathbf{r},t) = \mathrm{e}^{-\mathrm{i}\omega t + \mathrm{i}\mathbf{k}\cdot\mathbf{r}} \int_{-\infty}^{t} \mathrm{d}t' \int \mathrm{d}^3 v P(\mathbf{v}) g\left[\mathbf{r} + \mathbf{v}(t'-t), \mathbf{v}, t\right] \times \mathrm{e}^{\mathrm{i}(kv_z - \omega)(t'-t)} + \mathrm{c.c.} \qquad (3)$$

where $P(\mathbf{v})$ is in general a polynomial formed from the products of various velocity components (*e.g.*, P is a zeroth order polynomial in the case of the density; $P(\mathbf{v}) = \mathbf{v}$ in the case of the fluid velocity, and so on).

Due to the localized character of the $\psi(\mathbf{r},t)$ function, $g(\mathbf{r},\mathbf{v},t)$ is different from zero only if its spatial argument lies within the envelope of the wave-packet. In the spatial regions well outside the wave-packet, we can estimate $t' - t \approx r_\perp / v_\perp$, where the subscript $\perp$ refers to the *x, y* directions and $r_\perp \gg L$ is the distance of the observation point from the packet itself. L represents the typical spatial scale of the amplitude $\psi(\mathbf{r},t)$. For the electrons with $v_\perp \approx v_{\mathrm{te}} = \sqrt{2T/m}$, the exponential in the integrand of equation (3) is a rapidly oscillating function of v_z, with the period of the order of $v_{\mathrm{te}}/kr_\perp \ll v_{\mathrm{te}}$. Therefore, at $v_\perp \approx v_{\mathrm{te}}$, the integration over v_z does not produce any appreciable contribution. The v_z-integral is not exponentially small only for very large $v_\perp$-values, $v_\perp > v_{\mathrm{te}} k r_\perp \gg v_{\mathrm{te}}$. However for such large values of $v_\perp$, the unperturbed e.d.f. f_0 entering equation (2) is exponentially small by itself. We can conclude that at the first order in the i.a. wave amplitude the moments of the e.d.f. are exponentially small outside the region occupied by the wave-packet.

The present procedure is based on the linearization of the Vlasov equation which is permissible provided the electron trapping in the wave potential troughs is prevented by a small amount of collisionality [10]. Dealing with slowly propagating, transversely localized wave-packets, it is not expected that the bounce motion of trapped electrons could cause any significative departure from the Maxwellian distribution since in the resonant region of the velocity space, *i.e.* for $v_z \approx v_\varphi \approx c_{\mathrm{s}} \ll v_{\mathrm{te}}$, the equilibrium electron distribution function is almost flat. Possible effects of trapped ions [1,5] are not considered since in the spirit of the present analysis the ion acoustic wave-packet is a non-evolving drive for the "halo" formation.

A finite value of the moments of the e.d.f. outside the wave-packet can be found at higher orders. The kinetic equation for the second order perturbation of the e.d.f. f_2 reads

$$\frac{\partial f_2}{\partial t} + \mathbf{v} \cdot \frac{\partial f_2}{\partial \mathbf{r}} \approx -\frac{e}{m} \frac{\partial \varphi}{\partial \mathbf{r}} \cdot \frac{\partial f_1}{\partial \mathbf{v}} \equiv H(\mathbf{r},\mathbf{v},t), \qquad (4)$$

where the term $(-e/m)(\partial\varphi_2/\partial\mathbf{r})(\partial f_0/\mathbf{v})$ has not been considered for the moment. The l.h.s. of equation (4) being a linear differential operator, it corresponds to neglect, in the forthcoming evaluation of n_2, the contribution of that part of f_2 proportional to φ_2. Physically this term describes the plasma neutrality at higher orders and its effect will be incorporated in our treatment, at the right order, later. The solution of equation (4) reads

$$f_2(\mathbf{r},\mathbf{v},t) = \int_{-\infty}^{t} \mathrm{d}t' H\left[\mathbf{r} + \mathbf{v}(t'-t), \mathbf{v}, t'\right], \qquad (5)$$

where the function $H(\mathbf{r},\mathbf{v},t)$, quadratic in the wave field amplitude, contains terms fluctuating at twice the wave frequency ω, as well as steady contributions. Let us assume to follow the evolution of the wave-plasma system over times shorter than both the inverse of the collisionless damping rate [9], $t \ll \tau_{\mathrm{lin}} = \sqrt{M/m}/\omega_{\mathrm{pi}}$, and the dispersive spreading time [11], $t \ll \tau_{\mathrm{disp}} = k^2L^2/\omega_{\mathrm{pi}}$. Moreover, the condition $kL \ll M/m$ guarantees that the transit time of a thermal electron is much shorter than the damping time. Under these assumptions we can assume that the dependences of the field envelope on $\mathbf{r}$ and t act through their combination $\mathbf{R} \equiv \mathbf{r} - \mathbf{v}_{\mathrm{g}} t$. Then, the function $\psi(\mathbf{R})$ follows a pure translational motion of the wave-packet, with the group velocity $\mathbf{v}_{\mathrm{g}}$, $\mathbf{R}$ being the spatial coordinate in the reference frame of the packet. Accordingly, it is worth defining the wave frequency $\Omega \equiv \omega - \mathbf{k}\cdot\mathbf{v}_{\mathrm{g}}$, and the electron velocity $\mathbf{V} \equiv \mathbf{v} - \mathbf{v}_{\mathrm{g}}$. Therefore, in the comoving reference frame, the second order stationary part of the e.d.f. reads

$$f_2^{\mathrm{st}}(\mathbf{R},\mathbf{V}) = \frac{e^2k^2}{m^2} \int_{-\infty}^{0} \mathrm{d}\tau \psi^\star(\mathbf{R} + \mathbf{V}\tau) \times \frac{\partial}{\partial V_z} \left\{ \frac{\partial f_0}{\partial V_z} \int_{-\infty}^{0} \mathrm{d}\tau' \psi[\mathbf{R} + \mathbf{V}(\tau' + \tau)] \mathrm{e}^{\mathrm{i}(kV_z - \Omega)\tau'} \right\} + \mathrm{c.c.} \qquad (6)$$

We notice that $\mathbf{R}_\perp = \mathbf{r}_\perp$ and $\mathbf{V}_\perp = \mathbf{v}_\perp$. The superscript *st* will be omitted from now on.

3 Long range perturbation of the fluid moments

Let us evaluate the second order electron density perturbation $n_2(\mathbf{R})$ well outside the wave-packet, *i.e.* for $r_\perp \gg L$. We will show later that the main contribution to equation (6) comes from the electrons with their parallel velocities $V_z \ll v_\perp$. At large distance from the packet, at $r_\perp \gg L$, a characteristic value of τ in equation (6) can be derived by requiring that the argument of the outer integral be placed into the region occupied by the packet: then, $\tau \approx -r_\perp/v_\perp$, with possible deviations in the range $\pm L/v_\perp$. Let us consider now the inner integral, in τ'. Due to the localized character of the wave-packet, non zero contributions to this integral come solely from $|\tau'| \leq L/v_{\mathrm{te}}$.

By integrating equation (6) over V_z, we get

$$\int_{-\infty}^{+\infty} \mathrm{d}V_z f_2 \approx - \frac{e^2k^2}{m^2v_\perp^2}\int_{-\infty}^{+\infty}\mathrm{d}V_z \int_{-\infty}^{0}\mathrm{d}s\, s\left[\frac{\partial}{\partial Z}\psi^\star\left(\mathbf{r}_\perp + \hat{\mathbf{n}}s, Z + \frac{V_z}{v_\perp}s\right)\right] \times\left\{\frac{\partial f_0}{\partial V_z}\int_{-\infty}^{0}\mathrm{d}s'\,\psi\left[\mathbf{r}_\perp + \hat{\mathbf{n}}\left(s'+s\right), Z + \frac{V_z}{v_\perp}s\right]\mathrm{e}^{-\mathrm{i}qs'}\right\} + \mathrm{c.c.} \tag{7}$$

In equation (7) $\hat{\mathbf{n}} \equiv \mathbf{v}_\perp/v_\perp$, $q \equiv (\Omega - kV_z)/v_\perp$, and the new integration variables $s = v_\perp\tau$ and $s' = v_\perp\tau'$ have been introduced. Moreover, the term proportional to $s'V_z/v_\perp$ in the argument of the inner integrand has been deleted, according to the observations made just before equation (7). Finally, the Z- and the $\mathbf{r}_\perp$-dependences have been explicitly separated.

To be definite let us consider a Gaussian wave-packet $\psi(\mathbf{R}) = \psi_0\exp(-r_\perp^2/L_\perp^2 - Z^2/L_z^2)$ where ψ_0 is the real constant amplitude of the electrostatic potential, L_z and $L_\perp$ are the typical spatial scales of the electric field distribution along the Z-axis and in the (x, y) plane, respectively. While integrating over the transverse velocity, *i.e.* in $\mathrm{d}^2v_\perp = v_\perp\mathrm{d}v_\perp\mathrm{d}\theta$, the effective range of θ values which produces non negligible contributions turns out to be very small provided $r_\perp \gg L_\perp$, *i.e.* $\theta \approx L_\perp/r_\perp \ll 1$. Then, the exponents generated by the Gaussians can be approximated as follows: $(\mathbf{r}_\perp + \hat{\mathbf{n}}s)^2 \approx (r_\perp + s)^2 + s^2\theta^2$ and $(\mathbf{r}_\perp + \hat{\mathbf{n}}(s+s'))^2 \approx (r_\perp + s + s')^2 + (s+s')^2\theta^2$. The θ-integration is then straightforward. By further specifying the unperturbed e.d.f. to be an isotropic Maxwellian, $f_0(v_\perp, V_z) = n_0(m/2\pi T)^{3/2}\exp[-mv_\perp^2/2T - m(V_z + v_\mathrm{g})^2/2T]$, the second order density perturbation reads

$$n_2 \approx \frac{n_0}{2}\left(\frac{e\psi_0}{T}\right)^2\left(\frac{m}{T}\right)^{1/2}\frac{k^2L_\perp^3}{L_z^2}\int_0^\infty \frac{\mathrm{d}v_\perp}{v_\perp^2}\exp\left(\frac{-mv_\perp^2}{2T}\right) \times\int_{-\infty}^{+\infty}\mathrm{d}V_z(V_z + v_\mathrm{g})\left(Z - r_\perp\frac{V_z}{v_\perp}\right)\exp\left(-\frac{q^2L_\perp^2}{2}\right) \times\exp\left[-\frac{2}{L_z^2}\left(Z - r_\perp\frac{V_z}{v_\perp}\right)^2\right]. \tag{8}$$

The presence of the factor $\exp\left(-q^2L_\perp^2/2\right)$ shows that, indeed, only small V_z, *i.e.* $|V_z| < v_\mathrm{te}/kL_\perp$, contribute to the density perturbation.

The integration in V_z can be performed analytically and we remain with the $v_\perp$-integral to be performed numerically. Then equation (8) can be cast in the following form

$$\frac{n_2/n_0}{(e\psi_0/T)^2} \approx \frac{\pi^{1/2}}{2}\frac{\chi^2}{R^3\sigma^3}\int_0^\infty\frac{\mathrm{d}v}{v} \times\exp\left[-v^2 - \frac{(\chi\eta - \rho R\Lambda/v)^2}{2R^2\sigma^2}\right] \times\left[(v\xi + v_\mathrm{g})(\eta - \rho\xi)\sigma^2 - \frac{\rho v}{4}\right], \tag{9}$$

where the following dimensionless quantities have been defined $\rho = r_\perp/L_z$, $\eta = Z/L_z$, $R = L_z/L_\perp$, $\chi = kL_z$, $v = v_\perp/v_\mathrm{te}$, $\Lambda = L_\perp\Omega/v_\mathrm{te}$, $\sigma = \left[\rho^2 + (\chi/2R)^2\right]^{1/2}$, $\xi = \left[\eta\rho + (\Lambda\chi/4vR)\right]/\sigma^2$. A key parameter occurring in equation (9) is Λ, which represents the ratio between the time an electron with $v_\perp \approx v_\mathrm{te}$ needs to cross the packet and the wave period Ω^{-1}. Large Λ values correspond to the case in which the density perturbation is determined by the *resonant electrons*, that is by those electrons with $V_z \approx \Omega/k$, which remain in interaction with the waves for long. On the contrary, for small Λ's, the interaction has a nonresonant character and a large fraction of electrons contribute to it.

Figure 1 shows the shaded contours of the function defined in equation (9) in the (ρ, η)-plane, for $\chi = 20$, in three cases, $\Lambda = 1$, $R = 0.1$ (Fig. 1a), $\Lambda = 0.39$, $R = 0.3$ (Fig. 1b), $\Lambda = 0.1$, $R = 1$ (Fig. 1c). An H plasma has been considered. Due to the i.a. dispersion, in our dimensionless variables we have

$$v_\mathrm{g} = (\omega/k)/\left(1 + k^2\lambda_{\mathrm{D,e}}^2\right) = (\omega/k)/\left[1 + \left(\chi^2/2\right)\left(1/R^2\Lambda^2\right)\left(\Omega/\omega_{\mathrm{p,e}}\right)^2\right].$$

Going from Figure 1a to Figure 1c $L_\perp$ decreases while all other dimensional parameters remain constant. As it is shown, a finite amplitude density perturbation extends over several hundred times the typical scale of the packet, larger Λ values corresponding to larger perturbation amplitudes and to its wider (in $r_\perp$) extension. An analytical estimate of the behaviour of the "halo" at large distance from the packet may be found by calculating equation (9) for $\eta = 0$, *i.e.* for $Z = 0$. It gives

$$\left.\frac{n_2/n_0}{(e\psi_0/T)^2}\right|_{\eta=0} \approx -\frac{\pi}{16}\frac{\chi^2\rho}{R^3\sigma^3}\mathrm{e}^{-2\mu} \times\left[1 + 2^{1/2}\frac{\chi\sigma}{R\rho}\left(\frac{\chi\Lambda}{4R\sigma^2} + v_\mathrm{g}\right)\right], \tag{10}$$

where $\mu = \rho\Lambda/2^{1/2}\sigma$. In the limit $\rho \gg \chi/2R$ the density perturbation in the plane $\eta = 0$ is approximately

$$\left.\frac{n_2/n_0}{(e\psi_0/T)^2}\right|_{\eta=0} \approx -\frac{\pi}{16}\frac{\chi^2}{R^3\rho^2}\left(1 + \frac{\sqrt{2}\chi v_\mathrm{g}}{R}\right)\mathrm{e}^{-2^{1/2}\Lambda}, \tag{11}$$

that is it decreases with the power law ρ^{-2}. Moreover, from equation (11) we see that for given values of n_0, ψ_0, and T, n_2 is larger the larger is the number of wavelengths contained in the packet, and the fatter is the packet. Finally, the apparently non intuitive dependence on Λ is explained by the fact that the larger is Λ the more the "halo" thickens around the conic surface whose vertex angle is $\alpha \approx \pi/2 - \Lambda R/\chi$, measured from the Z-axis. Therefore it is worth stressing that the behaviour given by equation (10) does not necessarily represent the maximum (at a given $r_\perp$) density. Figure 2 displays the absolute value of the normalized density perturbation [see Eq. (9)] as a function of $r_\perp/L_\perp$, taken at $\eta = 0$, for $\Lambda = 0.3$ (Fig. 2a),

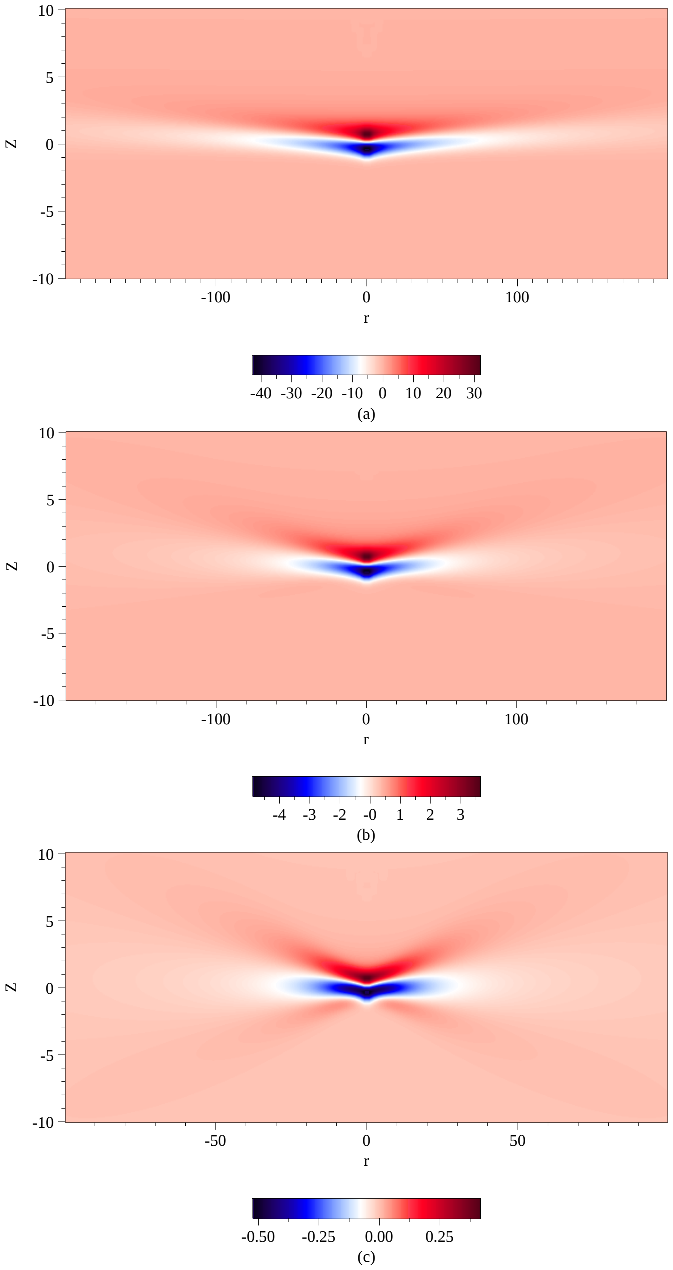

Fig. 1. The shaded contours of the spatial distribution of the second order electron density perturbation n_2, normalized to $n_0(e\psi_0/T)^2$, in the co-moving reference frame $r_\perp - Z$ (both lengths are normalized to L_z), in the case of a *very broad* (a), a *broad* (b), and a *spherical* (c) wave-packet. The relevant parameter values are: $\Lambda = 1$, $R = 0.1$ (a), $\Lambda = 0.39$, $R = 0.3$ (b), and $\Lambda = 0.1$, $R = 1$ (c), respectively. Other values common to the three cases are $\chi = 20$.

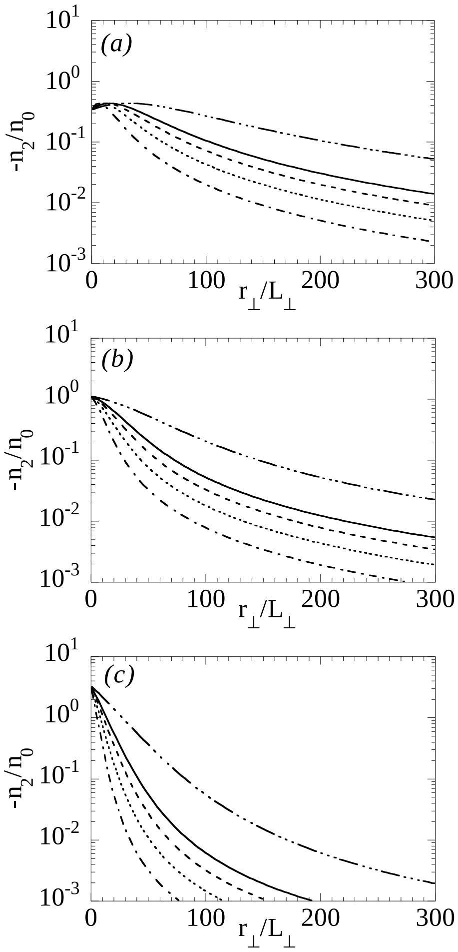

Fig. 2. The absolute value of the normalized density perturbation is plotted as a function of $r_\perp/L_\perp$, at $Z = 0$, for $\Lambda = 0.3$ (a), 1 (b), and 3 (c). The ratio between the longitudinal extension of the wavepacket and the wavelength is $\chi = 40$ (dot-dashed lines), 60 (dotted lines), 80 (dashed lines), 100 (full lines), and 200 (dot-dot-dot-dashed lines). Moreover $R = 1$.

1 (Fig. 2b), and 3 (Fig. 2c). Several values of the ratio between the longitudinal extension of the wavepacket and the wavelength have been considered: $\chi = 40$ (dot-dashed lines), 60 (dotted lines), 80 (dashed lines), 100 (full lines), and 200 (dot-dot-dot-dashed lines). The weakening of the long range perturbation in the plane $Z = 0$ with increasing Λ (going from Fig. 2a to Fig. 2c) is in agreement with the behaviour shown by equation (11) (see the discussion above). Observe that here, differently from Figure 1, the radial distances are normalized over $L_\perp$, which increases with Λ for constant electron temperature and wave frequency.

The time required for building up the "halo" at a distance $r_\perp$ can be estimated as $\tau_{\rm halo} \approx r_\perp/v_\perp \approx (r_\perp/L_\perp)\Lambda\omega_{\rm pi}^{-1}$. Then, in order the picture of the extended "halo" to be meaningful the following ratios should be large parameters:

(i) $\tau_{\rm lin}/\tau_{\rm halo} \approx (M/m)^{1/2}/[\Lambda(r_\perp/L_\perp)]$, that is $r_\perp/L_\perp < 4 \times 10^2$;
(ii) $\tau_{\rm disp}/\tau_{\rm halo} \approx \chi^2/[\Lambda(r_\perp/L_\perp)]$, that is $r_\perp/L_\perp < 4\times 10^3$, where the explicit figures refer to Figure 1.

Moreover, the inequality $\chi \ll M/m$ is easily well satisfied. Finally, notice that due to the role played by the ion mass in the typical time scales of the problem, large-mass ions help the formation of a quasistationary extended "halo".

4 Long-range electrostatic and magnetic fields

After demonstrating the existence of a long-range perturbation of the electron density around a localized i.a. wave-packet, we can go further into the analysis and determine the second order electrostatic potential φ_2 which is established in order to satisfy plasma quasineutrality conditions over scale lengths larger than the Debye radius, that are not consistently treated after equation (4). To this aim, let us consider n_2 as a "forcing term" in the electron density equation. From the Maxwell-Boltzmann law, we get the total electron density perturbation as $\delta n_e \approx n_0(e\varphi_2/T_e) + n_2$, which will be used to determine φ_2, once n_2 is known. To this aim, n_2 can be safely considered independent of φ_2, at the second order in the i.a. perturbation. By imposing the quasineutrality constraint, $\delta n_e = \delta n_i$, and introducing the response of the ions through their continuity and momentum equations, we obtain the inhomogeneous differential equation for the electrostatic potential

$$\nabla^2\varphi_2 - \frac{Mv_g^2}{T}\frac{\partial^2\varphi_2}{\partial Z^2} \approx \frac{v_g^2 M}{en_0}\frac{\partial^2 n_2}{\partial Z^2}\,. \tag{12}$$

Finally, we should mention that besides the electric charge perturbations an electric current density is also produced as a consequence of the appearance of f_2. In the laboratory frame it reads $\mathbf{j}_2^{\text{lab}} = \mathbf{j}_2 - e\mathbf{v}_g n_2$, where $\mathbf{j}_2$ is the "forcing current" in the co-moving frame. With a procedure conceptually similar to that outlined to calculate φ_2, the insertion of $\mathbf{j}_2$ into the Maxwell equations allows one to find the long-range magnetic field perturbation surrounding the i.a. wave-packet during its propagation through the plasma. Having addressed here the problem of the electromagnetic nature of the "halo" surrounding the i.a. wave-packet, the explicit calculation of the electrostatic potential perturbation and of the quasistationary magnetic field is left to a forthcoming extended paper. Here, we give a simple estimate of the effect of the propagation of a wave-packet with normalized amplitude $e\psi_0/T_e = 10^{-2}$, in a dense plasma, with $n_0 = 10^{18}$ cm^{-3} and $T_e = 100$ eV. By using the analytical evaluation of equation (11), we estimate the following perturbation intensities: the second order density perturbation as $|n_2/n_0| \approx (\pi/16)(\chi^2/R^3\rho^2)(e\psi_0/T_e)^2$, between 0.3% and 1% for $e\psi_0/T_e \approx 0.01$, $R = 0.1$, and $30 < \rho < 50$; the second order e.s. potential as $e\varphi_2/T_e \approx (v_g/c_s R)^2|n_2/n_0|$; finally, the induced quasistatic azimuthal magnetic field as $|B_\phi| \approx (\pi^2/4)(en_0 c_s/c)L_z(e\psi_0/T_e)^2(\chi^2/R^3)(\log\rho/\rho)$, that is, $|B_\phi(G)|/L_z(\mu\text{m}) \approx 1.6\times10^3(\log\rho/\rho)$ with $c_s = \sqrt{ZT_e/M}$. Then a wave-packet 3 μm long (that is about 50 times λ_{De}) is expected to produce a magnetic field of 200 G at $\rho = 100$. Notice that the present mechanism of magnetic field generation does not rely on the presence of an equilibrium temperature gradient, being the background plasma completely uniform.

5 Concluding remarks

In conclusion, the propagation of a well localized packet of low phase velocity waves turns out to be a physical process which affects the whole space around the wave-packet itself. Second order charge density and current density distributions are generated which are sources of quasistationary long-range electric and magnetic fields, whose amplitudes decrease in space typically with a power law. Besides the principal interest for the new scenario which is depicted by our analysis, new opportunities of diagnosing the presence of i.a. wave-packets by means of probes or, better, laser scattering techniques [8], without interfering with the packet itself, can be suggested.

Finally, the present analysis has been performed by assuming an unmagnetized and uniform background plasma, the scope of this work being the principle demonstration of the long range effects of the propagation of a localized ion-acoustic wave-packet. Of course, the evaluation of the "halo" effects in an actual experimental situation would need to take into account the consistent effect of the induced magnetic field back on the electron trajectories, as well as the effects of plasma gradients on the scales of the wavepacket dimension. We do not expect that these effects would alter the principle mechanism of the "halo" formation, however we leave the investigation of such issues for a forthcoming, more extended paper.

One of the authors (D.R.) is grateful to the Istituto di Fisica del Plasma "Piero Caldirola", C.N.R., for its hospitality. Useful discussions with J. D'Avanzo and I. Murusidze are acknowledged with pleasure.

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